Code : 101102

B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

(Calculus, Multivariable Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Answer/Choose the correct option of the following (any *seven*) : 2×7=14

(a) The sequence $\left(\frac{3}{(n!)^2}\right)$ is

(i) divergent

(ii) convergent

(iii) oscillatory

(iv) None of the above

(b) The function $f(x) = \begin{cases} x\sin\frac{1}{x} - 1, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ at $x = 0$ has a

(i) mixed discontinuity

(ii) continuity

(iii) removable discontinuity

(iv) None of the above

(c) Locus of the centre of curvature of a curve is called

(i) envelop of the curve

(ii) involute of the curve

(iii) evolute of the curve

(iv) None of the above

(d) The value of $\Gamma -\frac{5}{2}$ is

(i) $\frac{8\sqrt{\pi}}{15}$

(ii) $-\frac{\sqrt{8\pi}}{15}$

(iii) $-\frac{8\sqrt{\pi}}{15}$

(iv) None of the above

(e) Write the statement of Parseval's theorem.

(f) If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

(i) -1

(ii) 0

(iii) 1

(iv) 2

(g) Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{bmatrix}$ and let $\lambda_1 \geq \lambda_2 \geq \lambda_3$ be the eigenvalues of A. Then the triple $(\lambda_1, \lambda_2, \lambda_3)$ equals

(i) (9, 4, 2)

(ii) (8, 4, 3)

(iii) (9, 3, 3)

(iv) (7, 5, 3)

(h) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then A^{50} is

(i) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 0 & 0 \\ 50 & 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 48 & 0 & 0 \\ 48 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

(i) Define vector space.

(j) Define basis.

2. (a) State and prove the Cauchy mean value theorem. 8

(b) Evaluate $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$. 6

3. (a) Show that the evolute of a cycloid is another cycloid. 8

(b) Expand $\tan^{-1} x$ in power of $x - \frac{\pi}{4}$. 6

4. (a) Show that $\Gamma n \Gamma 1 - n = \frac{\pi}{\sin n\pi}$, $(0 < n < 1)$. 7

(b) Evaluate the integral $\int_0^1 x^4 (1 - \sqrt{x})^5 \, dx$. 7

5. *(a)* Find the Fourier series expansion of the following periodic function of period 4 :

$$f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 < x \le 2 \end{cases}$$

Hence, show that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots\cdots\cdots = \frac{\pi^2}{8}$$ 7

(b) Find the volume of the solid generated by revolving the region bounded by the curves $y = 3 - x^2$ and $y = -1$ about the line $y = -1$. 7

6. *(a)* Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 7

(b) For what values of k, the equations

$$x+y+z=1$$
$$2x+y+4z=k$$
$$4x+y+10z=k^2$$

have a solution? Solve them completely in each case. 7

7. *(a)* Reduce the matrix

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

to the diagonal form. 7

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(b) Let T be a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^3$, where $Tx = Ax$, $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}$ and $x = (x\ y)^T$. Find ker(T), ran(T) and their dimension. 7

8. *(a)* Let V be the set of all ordered (x, y), where x, y are real numbers. Let $\mathbf{a} = (x_1, y_1)$ and $\mathbf{b} = (x_2, y_2)$ be two elements in V. Define the addition as $\mathbf{a} + \mathbf{b} = (x_1, y_1) + (x_2, y_2) = (2x_1 - 3x_2, y_1 - y_2)$ and the scalar multiplication as $\alpha(x_1, y_1) = (\alpha x_1 / 3, \alpha y_1 / 3)$. Check whether V is a vector space or not. Explain the reason. 7

(b) Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ z+y \\ x+z \\ x+y+z \end{pmatrix}$$

Find the matrix representation of T with respect to the ordered basis

$$X = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

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in $\mathbf{R}^3$ and

$$Y = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

in $\mathbf{R}^4$. 7

9. (a) State and prove the rank-nullity theorem. 7

(b) Test the convergence of the following : 7

$$1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \frac{5!}{5^5} \cdots\cdots\cdots\cdots \infty$$

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