Code : 211101

B.Tech 1st Semester Exam., 2015

MATHEMATICS—I

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Answer any *seven* of the following questions : 2×7=14

(a) Find the sum and product of eigenvalues of

$$A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$$

(b) If $y=\sin^3 x$, then find the Nth derivative (y_n).

(c) What is the value of

$$D^n\left(\frac{1}{ax+b}\right)?$$

(d) Define homogeneous function with an example.

(e) Find the radius of curvature of the curve given by $y=e^x$ at $x=0$.

(f) Write the expansion of $\sin n\theta$.

(g) State Cayley-Hamilton theorem.

(h) Define orthogonal and unitary matrices.

(i) Find the order and degree of the differential equation $dy=(y+\sin x)dx$.

(j) For what value of x, the matrix

$$\begin{bmatrix}3-x & 2 & 2\\ 2 & 4-x & 1\\ -2 & -4 & -1-x\end{bmatrix}$$

is singular?

2. Verify Cayley-Hamilton theorem for the matrix

$$A=\begin{bmatrix}2 & -1 & 2\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

Hence, compute A^{-1}. 14

3. Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

Show that the equation is satisfied by A. 14

4. *(a)* Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(b) If λ is an eigenvalue of an orthogonal matrix, then show that $1/\lambda$ is also an eigenvalue. 7+7=14

5. *(a)* Evaluate

$$\int_0^{\pi} \frac{dx}{a + b\cos x}$$

where $a > 0, |b| < a$.

(b) If $y = \sin[\log(x^2 + 2x + 1)]$, then show that

$$(1+x)^2 y_{n+2} + (2n+1)(1+x)y_{n+1} + (n^2+4)y_n = 0$$

7+7=14

6. *(a)* Expand $e^x \log(1-y)$ in powers of x and y up to terms of third degree.

(b) Discuss maxima and minima of $x^3y^2(1-x-y)$. 7+7=14

7. Solve the following : 7+7=14

(i) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

(ii) $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$

8. *(a)* If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\delta}{\delta x} + \frac{\delta}{\delta y} + \frac{\delta}{\delta z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(b) Trace the curve $y^2(2a-x) = x^3$. 7+7=14

9. *(a)* Find the dimensions of a rectangular closed box of maximum capacity whose surface is given.

(b) Establish the relation between gamma and beta functions. Hence find the value of $\Gamma\left(\frac{1}{2}\right)$. 7+7=14

* * *