Code: 211101

B.Tech 1st Semester Exam., 2015

MATHEMATICS-I

Time: 3 hours

Full Marks: 70

Instructions:

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.
- 1. Answer any seven of the following questions:

2×7=14

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(a) Find the sum and product of eigenvalues of

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

- (b) If $y = \sin^3 x$, then find the Nth derivative (y_n) .
- (c) What is the value of

$$D^n\left(\frac{1}{ax+b}\right)^n$$

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- (d) Define homogeneous function with an example.
- Find the radius of curvature of the curve given by $y = e^x$ at x = 0.
- (f) Write the expansion of $\sin n\theta$.
- (g) State Cayley-Hamilton theorem.
- Define orthogonal and unitary matrices.
- Find the order and degree of the differential equation $dy = (y + \sin x)dx$.
- For what value of x, the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

is singular?

2. Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Hence, compute A-1

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d)

- (3)
- Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

Show that the equation is satisfied by A.

4. (a) Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

- If \(\lambda\) is an eigenvalue of an orthogonal matrix, then show that $1/\lambda$ is also an 7+7=14eigenvalue.
- Evaluate

$$\int_0^{\pi} \frac{dx}{a + b \cos x}$$

where a > 0, |b| < a.

(b) If $y = \sin[\log(x^2 + 2x + 1)]$, then show that

$$(1+x)^2 y_{n+2} + (2n+1)(1+x)y_{n+1} + (n^2+4)y_n = 0$$

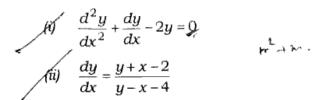
7+7=14

- Expand $e^x \log(1-y)$ in powers of x and y up to terms of third degree.
 - minima of Discuss maxima

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.7. Solve the following:



8. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show

$$\left(\frac{\delta}{\delta x} + \frac{\delta}{\delta y} + \frac{\delta}{\delta z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

- (b) Trace the curve $y^2(2a-x)=x^3$.
- Find the dimensions of a rectangular closed box of maximum capacity whose surface is given.
 - Establish the relation between gamma and beta functions. Hence find the value of $\Gamma\left(\frac{1}{2}\right)$. 7+7=14

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