Code : 211101

B.Tech 1st Semester Examination, 2016

Mathematics-I

Time : 3 hours *Full Marks : 70*

Instructions :

(i) There are Nine Questions in this Paper.

(ii) Attempt five questions in all.

(iii) Question No. 1 is Compulsory.

(iv) The marks are indicated in the right-hand margin.

---

1. Answer any seven of the following questions: 2×7=14

(a) Zero is a characteristic root of a matrix. if and only if matrix A is:

(i) non-singular matrix

(ii) singular matrix

(iii) symmetric matrix

(iv) none of above

(b) An $n \times n$ matrix is diagonalizable if and only if:

(i) it is singular matrix

(ii) it is symmetric matrix

(iii) it possesses n linearly independent Eigen vector

(iv) none of above

(c) The radius of curvature for the curve $s = \log(\tan\psi + \sec\psi) + \tan\psi \sec\psi$, where ψ is the angle which the tangent at any point to the curve makes with the x-axis is:

(i) $\sec^3 \psi$

(ii) $2\sec^3 \psi$

(iii) $3\sec^3 \psi$

(iv) none of above

(d) The value of $\left|-\frac{5}{2}\right.$ is

(i) $\frac{8\sqrt{\pi}}{15}$

(ii) $-\frac{\sqrt{8\pi}}{15}$

(iii) $-\frac{8\sqrt{\pi}}{15}$

(i) none of above

(e) If $u(x,y)=\left(\sqrt{x}+\sqrt{y}\right)^5$, then the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is

(i) $\frac{15}{4}u(x,y)$

(ii) $\frac{5}{2}u(x,y)$

(iii) $\frac{3}{2}u(x,y)$

(iv) none of above

(f) The value of $erfc(-x)$ is

(i) $1+erfc(x)$

(ii) 1–erfc(x)

(iii) 2+erfc(x)

(iv) 2–erfc(x)

(g) Find all the asymptotes of the curve $xy^2 = 4a^2(2a-x)$. 3

(h) Define similarity transformation.

(i) State Euler's Theorem for homogeneous function.

(j) Write the Abel's test for improper integral.

2. (a) Determine the rank of the given matrix A by reducing it in normal form 7

$$A=\begin{bmatrix}6 & 1 & 3 & 8\\ 4 & 2 & 6 & -1\\ 10 & 3 & 9 & 7\\ 16 & 4 & 12 & 15\end{bmatrix}$$

(b) For what values of λ and μ do the system of equations:

$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ 7

have (i) no solution (ii) unique solution (iii) more than one solution

3. (a) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ 7

Find the Eigen values of $3A^3 + 5A^2 - 6A + 2I - 0$

(b) Find A^4 with the help of Cayley Hamilton Theorem, if

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

4. (a) If $y=\cos(m \sin^{-1} x)$, then prove that $(1-x^2)y_2 - x y_1 + m^2 y = 0$. 7

(b) If $y = (x+\sqrt{1+x^2})^m$, then find $(y_n)_0$ 7

5. (a) Find the values of a and b in order that

$$\lim_{x \to 0}\left[\frac{x(1 - a\cos x) + b\sin x}{x^3}\right] = \frac{1}{3}.$$ 7

(b) If any tangent to the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ cuts off intercepts p and q from the axes, then find the value of $\frac{p}{a} + \frac{q}{a}$.

6. (a) Find the pedal equation of the parabola $y^2 = 4a(x+a)$. 7

(b) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $7\frac{13}{16}$. 7

7. Solve the following differential equations : 7+7=14

(a) $y'' + e^{2y}(y')^3 = 0$

(b) $(y + e^{1/x})dx - xdy = 0$

8. Let $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ be a second order differential equation. Let $a_0(x), a_1(x), a_2(x)$ be continuous and $a_0(x) \neq 0$ on an interval I and $y_1(x), y_2(x)$, be two linearly independent solutions. Show that the Wornskian of $y_1(x), y_2(x)$ satisfies the differential equation $a_0(x)W'(x) + a_1(x)W(x) = 0$. Also, show that the Wronskian is given by $W(x) = ce^{-\int [a_1(x)/a_0(x)]dx}$, where c is constant. 14

9. (a) Discuss the convergence of following improper integral 7

$$\int_1^\infty \frac{x\tan^{-1} x}{\sqrt{4+x^3}}\,dx$$

(b) Show that $\int_0^a \frac{1}{\sqrt[n]{a^n - x^n}} dx = \frac{\pi}{n} \operatorname{cosec}\left(\frac{\pi}{n}\right)$, where n>1. 7

***