Code : 211101

B.Tech 1st Semester Exam., 2017

MATHEMATICS—I

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*
(ii) *There are* ***NINE*** *questions in this paper.*
(iii) *Attempt* ***FIVE*** *questions in all.*
(iv) *Question No.* ***1*** *is compulsory.*

1. Choose the correct option/Answer any *seven* of the following : 2×7=14

(a) If the eigenvalue of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

is 6, then the eigenvalue of

$$A = \begin{bmatrix} 14 & -6 & 2 \\ -6 & 13 & -4 \\ 2 & -4 & 9 \end{bmatrix}$$

will be

(i) 6
(ii) $\frac{1}{6}$
(iii) 12
(iv) None of the above

(b) If

$$u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

then the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

is

(i) $\frac{1}{2}u$
(ii) $-\frac{1}{2}u$
(iii) $\frac{1}{2}\cot u$
(iv) $-\frac{1}{2}\cot u$

(c) If $u = u(y-z, z-x, x-y)$, then the value of

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

is

(i) 0
(ii) 1
(iii) 2
(iv) −1

(d) All the points of inflection of the function $f(x) = 2x^3 + 3x^2 - 36x$ are

(i) $x = 2, -3$

(ii) $x = -\frac{1}{2}$

(iii) $x = 0, \frac{1}{2}$

(iv) None of the above

(e) The function $f(x) = x^4 + x^2$ is

(i) concave

(ii) convex

(iii) either concave or convex

(iv) None of the above

(f) The value of

$$\frac{d}{dx}[erf\,(\alpha\, x)]$$

is

(i) $\frac{2\alpha}{\sqrt{\pi}} e^{-\alpha^2 x^2}$

(ii) $-\frac{2\alpha}{\sqrt{\pi}} e^{-\alpha^2 x^2}$

(iii) $\frac{\alpha}{\sqrt{\pi}} e^{-\alpha^2 x^2}$

(iv) None of the above

(g) All the asymptotes of the curve

$$y^2 (x-2)(x-3) - 9x^2 = 0$$

are

(i) $x = 3;\ y = \pm 3$

(ii) $x = 3;\ y = -3$

(iii) $x = 2, 3;\ y = 3$

(iv) $x = 2, 3;\ y = \pm 3$

(h) The order of the differential equation of all circles of given radius a is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(i) Write down the matrix of the given quadratic forms

$$2x^2 + 5y^2 - 6z^2 + 8xz - yz$$

(j) Define Wronskian of the solutions y_1, y_2, y_3 of the differential equation

$$a_0(x)\frac{d^3y}{dx^3} + a_1(x)\frac{d^2y}{dx^2} + a_2(x)\frac{dy}{dx} + a_3(x)y = 0$$

2. *(a)* Determine the rank of the given matrix A by reducing it in normal form

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ 7

(b) Show that the homogeneous system of equations

$$x + y\cos\gamma + z\cos\beta = 0$$
$$x\cos\gamma + y + z\cos\alpha = 0$$
$$x\cos\beta + y\cos\alpha + z = 0$$

has non-trivial solution if $\alpha + \beta + \gamma = 0$. 7

3. *(a)* If

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

then find the value of

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$ 7

(b) Using Cayley-Hamilton theorem, find A^{-1}, given that

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$ 7

4. *(a)* Find the nth derivative of $x^3 e^x \cos^3 x$. 7

(b) Expand $\log(\sin x)$ in power of $(x - a)$, where a is constant. 7

5. *(a)* Find the tangent at the point t on the curve $x = a\cosh t$, $y = b\sinh t$. 7

(b) Evaluate :

$$\lim_{x \to 0} \left[\frac{\log_{\sec x/2} \cos x}{\log_{\sec x} (\cos x/2)} \right]$$ 7

6. *(a)* Show that pedal equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 b^2}$$ 7

(b) Find the radius of curvature at any point t of the curve $x = a\cos^3 t$, $y = a\sin^3 t$. 7

7. Solve the following differential equations : 7+7=14

(i) $xy' = y^3 - x^3 - 3y^2 x + 3yx^2 + y$

(ii) $(3x^2 y^3 e^y + y^3 + y^2)\, dx + (x^3 y^3 e^y - xy) = 0$

8. *(a)* Solve : 7

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

(b) Find the value of

$$\int_0^\infty e^{-x^2}\, dx$$ 7

9. *(a)* Evaluate the following improper integral, if exist

$$\int_0^3 \frac{1}{3x - x^2}\, dx$$ 7

(b) Evaluate the integral

$$\int_0^\infty \frac{e^{-ax}\sin x}{x}\, dx,\ a > 0$$

and hence find the value of integral

$$\int_0^\infty \frac{\sin ax}{x}\, dx$$ 7

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