Code : 102102

B.Tech 1st Semester Exam., 2019
(New Course)

MATHEMATICS—I

(Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Answer/Choose the correct option of the following (any *seven*) : 2×7=14

(a) The envelope of the normals of the curve is called

(i) involute of the curve

(ii) evolute of the curve

(iii) the locus of the tangent of the curve

(iv) None of the above

(b) Define beta and gamma functions.

(c) If $f(x) = e^{-x}$ and $g(x) = e^{x}$, then in the Cauchy mean value theorem the value of c is

(i) geometric mean of a and b

(ii) $\frac{a+b}{ab}$

(iii) $\frac{a+b}{2}$

(iv) $\frac{ab}{a+b}$

(d) Which of the following options satisfies the Cauchy mean value theorem?

(i) $\sin x$ and $\cos x$ in the interval $[0, \pi]$

(ii) $\log_e x$ and $1/x$ in the interval $[1, e]$

(iii) $\log_e x$ and $1/x$ in the interval $[0, 1]$

(iv) $(x+2)^3(x-5)^2$ and $(x-5)^{1000}$ in the interval $[0, 6]$

(e) The series $\sum \frac{1}{n^p}$ is convergent, if

(i) $p < 1$

(ii) $p > 1$

(iii) $p \leq 1$

(iv) $p \geq 1$

(i) Let $N = \begin{bmatrix} 3/5 & -4/5 & 0 \\ 4/5 & 3/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then N is

(i) non-invertible

(ii) skew-symmetric

(iii) symmetric

(iv) orthogonal

(j) Define eigenvalue and eigenvector.

2. (a) Show that the equation to be evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$. 7

(b) Evaluate the integral $\int_0^1 (x \log x)^3 \, dx$. 7

3. (a) Show that

$$\int_0^1 \sin^m \theta \cos^n \theta \, d\theta = \frac{\left|\frac{m+1}{2}\right| \left|\frac{n+1}{2}\right|}{2\left|\frac{m+n+2}{2}\right|}$$ 7

(Continued)

(b) Find the area of the region that lies inside the circle $r = a\cos\theta$ and outside the cardioid $r = a(1 - \cos\theta)$. 7

4. (a) Discuss applicability of Rolle's theorem to the function $f(x) = |x|$ in $[-1, 1]$. 6

(b) Obtain the fourth degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$. Find the maximum error when $0 \le x \le 0 \cdot 5$. 8

5. (a) A given quantity of metal is to be cast into a half-cylinder, i.e., with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : \pi + 2$. 7

(b) Test the convergence of the series

$$\sum_{n=0}^{\infty} \frac{\sin^n |z\pi|}{n(n+1)}$$ 7

(Turn Over)

6. *(a)* Find the shortest distance between the lines $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. 7

(b) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^2$ at point (1, 2, 3) in the direction of $3i + 4j - 5k$. 7

7. *(a)* Prove that

$$\text{curl}(f\,\mathbf{v}) = (\text{grade } f) \times \mathbf{v} + f \text{ curl } \mathbf{v}$$

where f is a scalar function. 7

(b) Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^2 + y^2}{|x| + |y|} & , \ (x, y) \neq (0, 0) \\ 0 & , \ (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0) but its partial derivatives f_x and f_y do not exist at (0, 0). 7

8. *(a)* Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$ 7

(b) Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$ and $5x + 6y + 7z = c$ do not have a solution unless $a + c = 2b$. 7

9. *(a)* Examine whether the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

is diagonalizable. If so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. 7

(b) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ 7

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