Code : 103102

B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

(Calculus and Differential Equations)

Time : 3 hours

Full Marks : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are **NINE** questions in this paper.*

(iii) *Attempt **FIVE** questions in all.*

(iv) *Question No. **1** is compulsory.*

1. Choose the correct answer of the following (any *seven*) : $2 \times 7 = 14$

(a) The maximum value of $\sin x + \cos x$ is

(i) 1

(ii) 2

(iii) $\sqrt{2}$

(iv) 0

(b) The value of the integral

$$\int_C \{yz\,dx + (xz+1)\,dy + xy\,dz\}$$

where C is any path from $(1, 0, 0)$ to $(2, 1, 4)$ is

(i) 6

(ii) 7

(iii) 8

(iv) 9

(c) The series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots$$

is

(i) convergent

(ii) divergent

(iii) absolutely convergent

(iv) None of the above

(d) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(i) $\tan 2u$

(ii) $\cos 2u$

(iii) $\sin 2u$

(iv) $\cot 2u$

(Continued)

(e) If $A(x, y, z) = x^2 zi - 2y^3 z^2 j + xy^2 zk$, then $(\nabla \cdot A)$ at the point $(1, -1, 1)$ is

(i) 3

(ii) −3

(iii) 4

(iv) −4

(f) The value of $\iint dxdy$ over the region $9x^2 + 4y^2 \le 4$ is

(i) π

(ii) 2π

(iii) $\frac{\pi}{3}$

(iv) $\frac{2\pi}{3}$

(g) If P_n is the Legendre polynomial of first kind, then the value of $\int_{-1}^{1} xP_n P_n' dx$ is

(i) $\frac{2}{(2n+1)}$

(ii) $\frac{2n}{(2n+1)}$

(iii) $\frac{2}{(2n+3)}$

(iv) $\frac{2n}{(2n+3)}$

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(h) If J_n is the Bessel's function of first kind, then the value of $J_{-\frac{1}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}}\sin x$

(iv) $\sqrt{\frac{2}{\pi x}}\cos x$

(i) For the differential equation $t(t-2)^2 y'' + ty' + y = 0$, $t = 0$, is

(i) an ordinary point

(ii) a branch point

(iii) an irregular point

(iv) a regular singular point

(j) The solution of the equation $xp^3q^2 + yp^2q^3 + (p^3 + q^3) - zp^2q^2 = 0$ is $z =$

(i) $ax + by + (ab^{-2} + ba^{-2})$

(ii) $ax - by + (ab^{-2} - ba^{-2})$

(iii) $-ax + by + (-ab^{-2} + ba^{-2})$

(iv) $ax + by - (ab^{-2} + ba^{-2})$

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2\. (a) Find the evolutes of the curve

$$x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$$

where a is the constant.

(b) Show that

$$\Gamma(1/4)\cdot\Gamma(3/4)=2\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta=4\int_0^{\infty}\frac{x^2}{1+x^4}\,dx=\pi\sqrt{2}$$

7+7=14

3\. (a) Expand $f(x, y)=x^2y+3y-2$ in the powers of $(x+2)$ and $(y-1)$ by Taylor's theorem.

(b) Find the maxima and minima, if any, of

$$f(x)=\frac{x^4}{(x-1)(x-3)^3}$$

7+7=14

4\. Explain the Dirichlet conditions. Find out the Fourier series for the periodic function defined by

$$f(x)=\begin{cases}0, & -\pi\leq x<0\\ x^2, & 0\leq x<\pi\end{cases}$$

and hence find the sum of the series—

(a) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots$;

(b) $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}+\cdots$.

14

5\. (a) If $u=\log(x^3+y^3+z^3-3xyz)$, then find

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u$$

(b) Find $\nabla\cdot\nabla\varphi$, where $\varphi=xy^2z^4$. 7+7=14

6\. (a) Evaluate the integral by changing the order of integration

$$\int_0^{\infty}\int_0^{x} xe^{-\frac{x^2}{y}}\,dy\,dx$$

(b) Solve the differential equation

$$(x^2+y^2+x)\,dx-(2x^2+2y^2-y)\,dy=0$$

7+7=14

7\. Verify the Stokes' theorem for

$$A=(y-z+2)i+(yz+4)j-xzk$$

where S is the surface of the cube $x=0$, $y=0$, $z=0$, $x=2$, $y=2$ and $z=2$ above the xy-plane. 14

8\. **Solve in series, using Frobenius method, the equation** $x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$. 14

9. (a) State and prove orthogonal properties of Legendre polynomials.

(b) Find the complete integral of the equation $(p^2 + q^2)y = zq$. 7+7=14

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